MATHEMATICAL MODELS AND NEW ALGORITHMS
FOR IMAGE PROCESSING

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In this thesis mathematical models and techniques for processing two-dimensional signals (images) are developed, and new algorithms are proposed for solving important problems in image processing such as: Blind Source Separation - BSS, noise reduction and pattern recognition. As an introduction, in Section 2.1, main properties of the Principal Component Analysis - PCA method, also known as Karhunen-Loeve Transformation - KLT, are described, together with the analysis of related methods as: Independent Component Analysis - ICA and Projection Pursuit - PP. A detailed theoretical analysis is made, based on elements of the Shannon information theory, stating the connections among: Gaussianity, entropy, mutual information and independence of variables. In Section 2.2, classification methods for vectors in $\mathbb{R}^n$ by using minimum Euclidean distances are described, where orthonormal bases and orthogonalization methods are very important. It is shown that KLT, on which the eigenfaces technique for pattern recognition is based, is the optimal method. In Section 2.3, random field theories are described allowing to model pixel interactions in images, which includes the Ising model and Gaussian Random Field’s -GRF’s.

In chapters 3 and 4 new theoretical results are introduced, and new algorithms are developed with applications to image processing as presented in Chapter 5. As a generalization of ICA, in Chapter 3, blind separation of statistically dependent sources (Dependent Component Analysis - DCA) is approached, which is a new field of research with few previous works in the literature and with important applications to engineering. A new DCA algorithm is proposed, namely MaxNG, which allows the separation of dependent sources (Section 3.4). New fundamental theoretical results are presented, providing a sufficient condition on the sources that guarantees their separability through the maximization of Non-Gaussianity (NG) (Section 3.3). An NG measure based on the $L^2(\mathbb{R})$ distance is proposed, a non-parametric Parzen estimator is used to estimate the source probability density functions (pdf’s) (Section 3.2) and new approximation techniques are proposed to compute the NG measure in an optimized way through the Fast Fourier Transform - FFT (Section 3.4). Additionally, the robustness of MaxNG in noisy environments is analysed (Section 3.6) and a solution for the scale factor indeterminacy is proposed (Section 3.7).

In Chapter 4, motivated by the noise problem in astrophysical images, the reduction of Additive Gaussian White Noise - AGWN in sources estimated by MaxNG is analyzed (Section 4.2). A new Gaussian model is proposed, namely the Long Correlation - Gaussian Random Field (LC-GRF), which allows to model long range interactions existing in Cosmic Microwave Background - CMB images. Through a detailed analysis of the model properties (Section 4.3), new techniques for parameter estimation are developed based on the Maximum Likelihood - ML criterion and the Expectation - Maximization (EM) algorithm (Section 4.4). Additionally, the optimal Wiener filter is derived for the restoration of CMB images distorted with AGWN (Section 4.5).
In Chapter 5, the following applications to image processing are presented:

i) Classification of *Mirounga Leonina* images (Section 5.1);

ii) Corregistration of remote-sensed images (Section 5.2);

iii) Sub-pixel analysis of hyper-spectral remote-sensed images (Section 5.3);

iv) Source separation and noise reduction in astrophysical images (Section 5.4).

Following, the novel contributions included in this thesis are mentioned, including the corresponding publication references:

- A new algorithm for DCA, called MaxNG, was developed (Chapter 3). Theoretical results are provided justifying the method and providing an efficient implementation of MaxNG, in terms of its computational complexity and under the influence of additive Gaussian noise. Furthermore, an experimental analysis of the MaxNG performance in different scenarios is provided, by using simulated data as well as real data, and comparing it with traditional ICA solutions [29, 30, 32, 33, 35].

- A new model of GRF with long correlations was introduced (LC-GRF) and new techniques of parameter estimation were developed based on it, also, a noise reduction filter was designed (Chapter 4) [34].

- A novel algorithm for the classification of face images of *Mirounga Leonina*, using properties of the KLT representation (Section 5.1) [26, 27].

- A new algorithm for automatic corregistration of remote-sensed images was developed by using the KLT representation, which was applied to radar images (Section 5.2) [28].

- A new method for sub-pixel analysis of hyper-spectral images (Spectral Unmixing) based on MaxNG was proposed for the estimation of the percent contributions per pixel of several materials present in an image (water, vegetation, bricks, etc.) (Section 5.3). Experiments on simulated and real images are provided, showing the usefulness of the method and comparing it with traditional techniques based on ICA [33, 35].

- A new algorithm, called Minimax Entropy, based on MaxNG was developed, which was especially designed for the blind separation of astrophysical images (Section 5.4). Experiments on simulated images are provided showing the usefulness of the method and comparing it with traditional techniques based on ICA [32].

- The LC-GRF model was used for modelling the long correlations existing in Cosmic Microwave Background - CMB images and used for the development of a noise reduction Wiener filter (Section 5.4) [34].